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Sensitivity of a vehicle lane change control system to disturbances and measurement signal errors – Modeling and numerical investigations

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ABSTRACT

Automatic control of the lane change process forms a basis to automate more complex vehicle maneuvers. The lane change controller model presented in the paper has a mixed structure. In the open-loop structure, the controller works as a reference signals generator supplying a time course of the steering system angle (reference input signal), and two time courses of the lateral and angular shifts of the vehicle (reference output signals). In the closed-loop structure it works as a steering signal corrector (regulation based on the error signals between the reference and measured signals expressing vehicle trajectory). For the validation of the controller algorithm, extensive simulation investigations have been carried out. In these investigations, as a virtual control object, a very detailed and experimentally verified mathematical model of a medium-duty truck has been used. The control system model and its simulation investigations were already presented in several papers by the Authors. This paper presents the results of the research focused on the sensitivity of the control system to disturbances and measurement signal errors (noises, offsets, delays).

1. Introduction

Automation of lane change processes is a fundamental scientific and technological problem for advanced driver assistance systems (ADAS), autonomous cars and intelligent transport systems (ITS). It should be remembered that more complex manoeuvres (e.g. avoiding obstacles or overtaking) are actually sequences of elementary lane changes controlled in accordance with sophisticated scenarios. Due to the specific characteristics of the car's lateral dynamics, and because of variable and accidental traffic situations, the lane change process is difficult to control. This is especially visible when the car travels at high speed on a slippery road and in the presence of other road users. In such extremely difficult and risky conditions, only experienced racing drivers are able to do it (and not always). Lane changing manoeuvres have a fundamental impact on macroscopic and microscopic characteristics of traffic flows due to the interference effect they have on surrounding vehicles. So the issues of automating lane change processes seem to be an attractive multidisciplinary challenge for specialists in the

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field of road transport, vehicle dynamics, control theory, mechatronics, etc., and they are the subject of many studies. This statement results from the analysis of numerous review type publications focusing on ADAS, autonomous vehicles and ITS, e.g. [1–10].

Typical reported studies relate to controlling the movement of a car treated as a single input system. In this case the control process is realized through its 2WS-type (two-wheel) steering system directly affecting the vehicle lateral dynamics. Noteworthy, there are publications in which the car treated as a controlled object is a multiple input system. This happens when the car is equipped with a 4WS-type (four-wheel) steering system [11–13], or when the control of the vehicle's trajectory is supplemented by the traction and/or braking systems' operations [12–16]. In this article we will focus on the fundamental issues related to the control of the vehicle movement by the 2WS automated steering system.

1.1. Literature review – Publications by others

Literature on the lane change control by 2WS automated steering systems is quite extensive. Because of the subject and scope of this paper the literature review is limited to selected publications, focusing on the papers related to theoretical aspects of the vehicle lane change control systems as well as with the problems of sensitivity analysis to disturbances and measurement errors.

Development of control algorithms for lane change processes can be realized in various ways. First, using heuristic mathematical models describing driver operations in the vehicle-driver system, or in a more formalized way, based on general control theory. Driver models verified in many experimental studies can form the basis of effective fuzzy logic controllers. So it is rather surprising that the latter approach (based on the control theory) definitely prevails in publications. The control methods used in systems of automatic driving are based on both classic control theory (predictive and tracking systems with universal regulators) and modern control theory (predictive and tracking systems with robust regulators or other sophisticated controllers). Application of the optimal control theory is also visible.

The concept of predictive and tracking system with feedback (e.g. [17–22]) is especially visible in the lane change controlling. In this case, the controller includes a generator of reference signals defining the prescribed vehicle motion in the road plane and appropriate regulators correcting the reference control signal so that the actual vehicle trajectory is close to the reference trajectory. Correction of the steering signal by regulators' feedback actions is necessary for many reasons such as parametric uncertainties and unmodeled dynamics of the reference lane change model, and because of the disturbances and errors in signals monitoring the movement of the real object.

In many publications (e.g. [18,23–25]) an assumption is made a priori that the reference vehicle trajectory is a composition of elementary functions such as sinusoid or circle segments, parabolas or generally of the so-called Bezier curves. Trajectory planning is then regarded as a task of parametric optimization for heuristically assumed shape functions. Such optimization is conducted with regard to the smoothness of the trajectory, the time duration of the maneuver and the goal not to exceed the limits on the lateral acceleration and the angular velocity of the vehicle. It is possible to create the reference signals that ensure the balance between the ride comfort and handling and guarantee safety conditions over a specified period of time.

A series of papers contains also information on the regulation algorithms. Typically, classic linear PID regulators, either analog or digital, are used [26]. Most recently, an interest can be seen in solutions elaborated for adaptive regulators [27] and robust-type controllers based on the μ-Synthesis concept [28] or on the Model Predictive Control (MPC) method [17].

The optimal control theory (by Bellman and Pontryagin [29]), as well as parametric optimization of control systems has been applied in vehicle dynamics studies [30–32] and also in the optimization of lane change maneuvers. The paper [33] describes the optimal control strategies for hazard maneuvers involving lane changes. Minimization of the lane change time is presented in [34]. The use of the Linear Quadratic Regulator method (LQR method derived by Kalman directly from the general optimal control theory) is presented in [35].

All the lane change control systems have close-loop forms corresponding to the vehicle-driver scheme and their operation algorithms result from the mathematical models used to describe the lateral dynamics of vehicle movements in the road plane.

Literature analyses indicate that in the synthesis of the control algorithms, only very simplified models describing the lane change process are useful (especially important feature for the control algorithm's synthesis). Usually the car lateral dynamics is expressed by the well known "bicycle model" (extensively described in [36,37]). Its main advantage is the simplicity of description (the model is based on few variables and parameters) and sufficient accuracy. Note that this model contains parameters that depend not only on the design parameters of the vehicle, but also on the operating conditions (speed, wheels-road slip coefficients, etc.), so the on-line identification and estimation procedures might be necessary. Such on-line computations can be implemented directly in the control algorithms using so called Kalman filters [38].

It is very important for the satisfactory results of the automation of the lane change maneuver to take into account the sensitivity of the control system to the variations of vehicle model parameters (and even to its structure). Although the general theory of sensitivity of dynamic systems is already very well developed [39] and used to vehicle lateral dynamics investigations, its applications in the analysis of the lane change process are rather modestly documented in publications. An important issue concerning such sensitivity studies has been raised in the article [40]. Over there, the process of the double lane change was investigated in the presence of an excessive backlash and friction in the vehicle steering system, which was not included in the synthesis of the control algorithm. After exceeding certain clearance and friction values, the car could not be controlled.

Sensitivity analysis concerning signal disturbances and measurement errors seems to be very important in lane change control systems. It is well known that sensors and other mechatronic components of real control systems can be a source of serious interferences and faults (noise, offset, delay) [41]. Especially pernicious (possibly causing instability) are delay effects in the feedback control systems. Literature on universal control systems with delay is very rich (an extensive lists is included in [42]). For synthesis of control algorithms that are dedicated to systems with delay special methods have been elaborated. Generally, they are based on the concept of state prediction in the control system. But in many cases, if the time delay is small enough, a control with neglected delay effects appears satisfactory enough to ensure system stability and relatively acceptable control action. Publications discussing the impact of signal disturbances and errors on the effect of traffic control are relatively few, e.g. [43–45]. Unfortunately, articles discussing such sensitivities in lane change control systems focus more on problems resulting from communication imperfections between vehicle drivers than on imperfections in the operation of their internal automatic controllers, e.g. [46].

1.2. Literature review – Authors' publications and their originality

Authors' publications on the lane change control system and its sensitivity analysis include a number of journal articles and conference papers. This article lists only the most significant works [47–53].

The original concept of the lane change control system is based on the theory of optimal control implemented to the most reduced version of the bicycle model describing the lateral dynamics of the vehicle. Such simple dynamic model (here the reference model) enables analytical calculation of reference signals as well as LQR algorithms for the regulators. Of course, this concept requires a lot of numerical investigations to analyse the sensitivity of the developed system.

Sensitivity simulation tests were conducted to investigate the effects of changes in vehicle parameters (masses, wheel-road friction coefficients, vehicle speed), and due to the simplification of the reference model (neglected inertia parameters in the steering system). The problem of signal disturbances (noise, offset, delay) was also addressed, separately for each of those factors.

This paper continues sensitivity investigations of the proposed control system. Here, the investigations concern the sensitivity of the lane change controller to complex disturbances caused by the noise as well as offset and delay effects occurring simultaneously. The article is an extension of the paper presented at the 15th International Conference "Dynamical Systems – Theory and Applications" DSTA 2019 [52].

2. Authors' concept of the lane change control system

The lane change process refers to two variables – displacement of the centre of mass $Y(t)$ and angular orientation $\Psi(t)$ of the car body in relation to the centre of mass trajectory. According to experiences of drivers as well as the optimal control theory the steering system signal (steering wheel angle signal) should be based on the "bang-bang" – type form (terminology of time-optimal systems) and the control process can be divided into two phases: transposition and stabilization, see (Figs. 1 and 2).

The initial mathematical model which describes the lane change process is based on the "bicycle model". This dynamical model is supplemented by equations which transform speed variables from a local coordinate system (attached to the car) into trajectory variables in a global system (attached to the road). Also, a simple mathematical description of the steering system action completes the model.

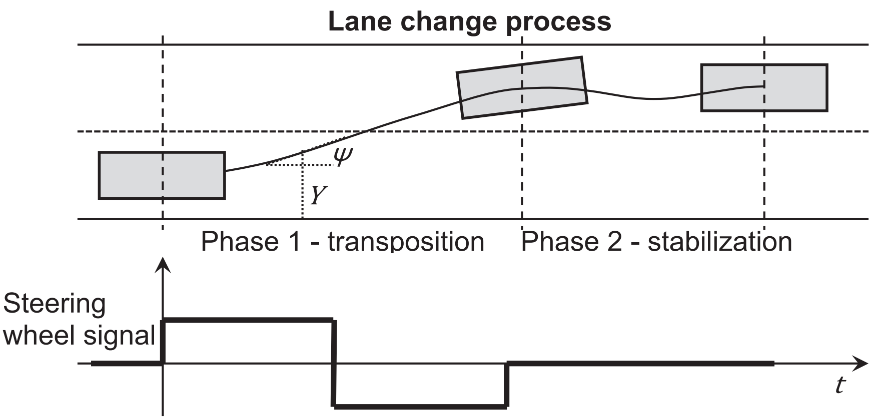


Fig. 1. The concept of time decomposition of the lane change control.

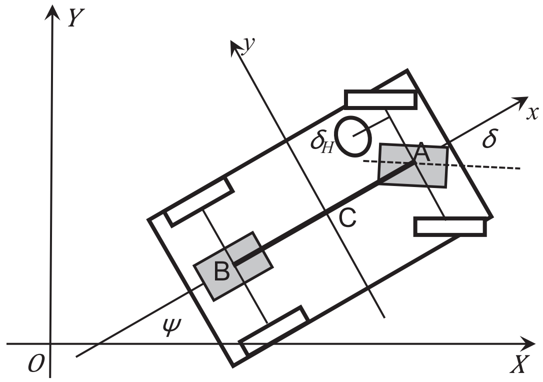


Fig. 2. The concept of the bicycle model of a vehicle.

Notations of variables and parameters of the model:
t – time (t = 0 denotes the moment of starting the control action),
δ_H – steering wheel angle,
δ – front wheel steer angle,
Ψ – vehicle yaw angle with respect to the road centre line,
Ω – yaw (angular) velocity of the vehicle $\left(\Omega(t) = \dot{\Psi}(t)\right)$,
U – lateral velocity of the vehicle in the local coordinate system,
X – position of vehicle centre of mass in the global coordinate system,
V – longitudinal velocity of the vehicle (constant) in the local coordinate system,
m – vehicle mass,
J – vehicle inertia moment with respect to the vertical axis passing through its mass centre,
a, b – distances from vehicle front and rear wheel axes, respectively, to the projection of the point representing the vehicle mass centre,
k_A, k_B – cornering stiffness for the centres of the front and rear wheel axes, respectively,
p – gain ratio of the steering system.
Equations of motions (mathematical bicycle model):

$$m\dot{U}(t) + \frac{k_A + k_B}{V}U(t) + \frac{mV^2 + k_A a - k_B b}{V}\Omega(t) = k_A\delta(t) \tag{1}$$

$$J\dot{\Omega}(t) + \frac{k_A a^2 + k_B b^2}{V}\Omega(t) + \frac{k_A a - k_B b}{V}U(t) = k_A a\delta(t) \tag{2}$$

Equations describing vehicle trajectory in the global coordinate system:

$$X(t) = \int_0^t \dot{X}(\tau)d\tau = \int_0^t (V\cos(\Psi(\tau)) - U(\tau)\sin(\Psi(\tau)))d\tau \tag{3}$$

$$Y(t) = \int_0^t \dot{Y}(\tau)d\tau = \int_0^t (V\sin(\Psi(\tau)) - U(\tau)\cos(\Psi(\tau)))d\tau \tag{4}$$

$$\Psi(t) = \int_0^t \Omega(\tau)d\tau \tag{5}$$

Equation describing the steering system:

$$\delta(t) = p\delta_H(t) \tag{6}$$

When the steering system angle $\delta_H(t)$ has a “bang-bang” form, the yaw angle $\Psi(t)$ is small enough and transformation equations can be linearized. After linearization and subsequent Laplace transformation one obtains the model in the s-operator domain. Using transfer functions $G_{\Psi\delta}(s)$ and $G_{Y\delta}(s)$ the model gets the following form:

$$Y(s) = G_{Y\delta}(s)\delta(s) \tag{7}$$

where

$$G_{Y\delta}(s) = \frac{VG_{\Omega\delta_0}\left(T_{Y\delta}^2 s^2 + 2\xi_{Y\delta}T_{Y\delta}s + 1\right)}{s^2\left(T_0^2 s^2 + 2\xi_0 T_0 s + 1\right)} \tag{8}$$

$$\Psi(s) = G_{\Psi\delta}(s)\delta(s) \tag{9}$$

where

$$G_{\Psi\delta}(s) = \frac{G_{\Omega\delta_0}(T_{\Omega\delta}s + 1)}{s\left(T_0^2 s^2 + 2\xi_0 T_0 s + 1\right)} \tag{10}$$

The transmittance parameters are non-linear functions of vehicle parameters $V, m, \ldots$ For example:

$$G_{\Omega\delta 0} = \frac{k_A k_B (a+b)V}{k_A k_B (a+b)^2 - mV^2(k_A a - k_B b)} \tag{11}$$

The transfer functions contain main as well as supplementary elements (which express only a subtle correction of the dynamics). By neglecting such components (formally by zeroing parameters $T_0, T_{\Omega\delta}$, and $T_{\Omega\delta}$), one obtains the reduced model with reduced transfer functions:

$$G_{Y\delta}(s) = \frac{VG_{\Omega\delta_0}}{s^2} \tag{12}$$

$$G_{\Psi\delta}(s) = \frac{G_{\Omega\delta_0}}{s} \tag{13}$$

This reduced model treated as the reference model is a basis for synthesis of all reference signals – input signal $\delta_{HR}(t)$ (bang-bang type waveform signal of the steering system angle) and corresponding signals $\delta_R(t)$, $Y_R(t)$ and $\Psi_R(t)$ (Fig. 3). Note, that signals' parameters T, δ_0, Y_0, Ψ_0 and vehicle's parameters are interrelated.

The lane change controller is composed by a reference signal generator and 2 regulators acting in a variable structure (Fig. 4).

The generator provides three reference signals $\delta_{HR}(t)$ (bang-bang type waveform signal of the steering system angle), $Y_R(t)$ and $\Psi_R(t)$ (waveform signals of the linear and angular vehicle positions computed for $\delta_{HR}(t)$ signal) which describe the lane change maneuver according to a simple reference model of the vehicle motion. The signals $Y_R(t)$, $\Psi_R(t)$ are set-point signals for two Kalman - type regulators which correct the real steering angle signal $\delta_H(t)$ to minimize errors between measured and desired waveforms of the variables. In the first phase of the control process, the transposition system is ON (activated) and the angular stabilization system is OFF (deactivated); in the second phase, these connections are reversed. The switching strategies can be also more sophisticated (e. g. earlier starting of the regulator 2 action).

When the switching strategy answers a two phase description of lane change process, the regulation equations take the forms:

$$\delta(s) = \delta_R(s) + \Delta\delta(s) \tag{14}$$

$$\delta_R(s) = p\delta_{HR}(s) \tag{15}$$

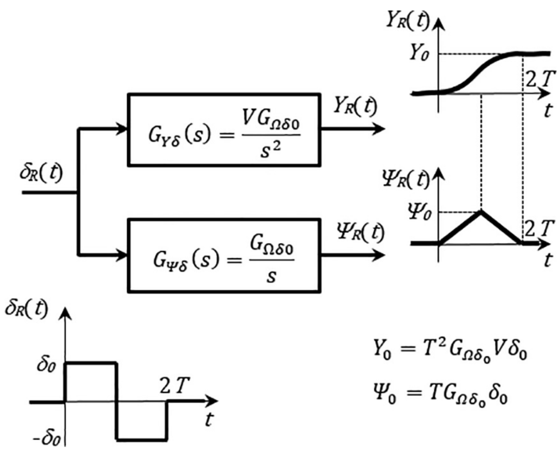


Fig. 3. The reference signals obtained from the reduced reference model.

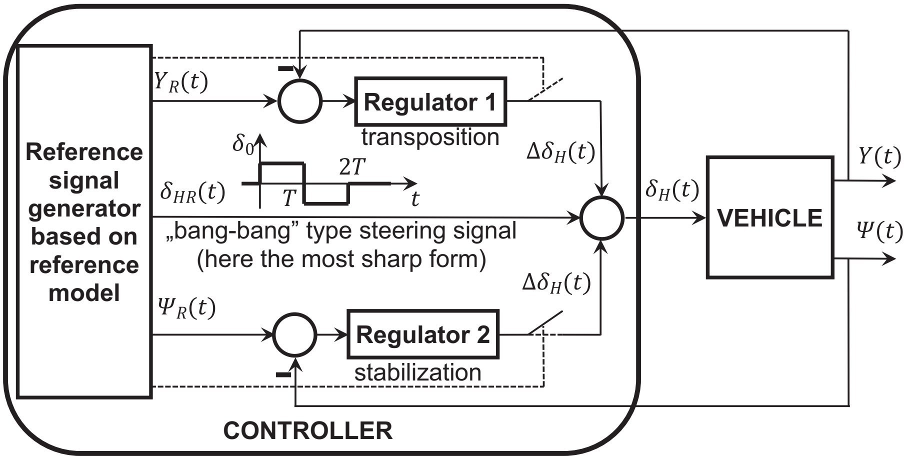


Fig. 4. Block diagram of the automatic control system.

For $t \leq 2T$ (transposition phase)

$$\Delta\delta(s) = R_Y(s)(Y_R(s) - Y(s)) \tag{16}$$

$$Y_R(s) = G_{Y\delta}(s)\delta_R(s) \tag{17}$$

For $t > 2T$ (stabilization phase)

$$\Delta\delta(s) = R_\Psi(s)(\Psi_R(s) - \Psi(s)) \tag{18}$$

$$\Psi_R(s) = 0 \tag{19}$$

Here, the Kalman-type regulators are described by the transfer functions $R_Y(s)$ and $R_\Psi(s)$. They are specified after solution of the standard LQR task (optimization task), and they have linear forms. Assuming the reduced model (12, 13) as the reference model they are simple standard regulators. For example: The regulator 1 is PD type, the regulator 2 is P type. So

$$R_Y(s) = K_{PD}(1 + T_{PD}s) \tag{20}$$

$$R_\Psi(s) = K_P \tag{21}$$

where K_{PD}, T_{PD}, K_P - parameters derived by analytic solution of the LQR task.
For example:

$$K_{PD} = \frac{\sqrt{p_{11}r}}{VG_{\Omega\delta_0}} \tag{22}$$

(here p_{11} and r – weight parameters in the LQR functional).

A special feature of the developed controller algorithm is an analytical linking of its parameters $(\delta_0, K_{PD}, \cdots)$ with parameters of the vehicle $(V, m, \cdots)$ and with parameters of the lane-change scenario (Y_0, Ψ_0, T). Thanks to the analytical forms of reference signals and regulators' algorithms, the automatic lane change controller can adapt its algorithm very quickly - in real time.

This authorial lane change control system was developed on a very simple idealized reference model. Therefore, to check its effectiveness in controlling a real vehicle, it is necessary to test its sensitivity, especially with regard to potential disturbances.

3. Disturbances in the control system – Theoretical approach

The conceptual control system is a basis for elaboration of real mechatronic system containing sensors, data processing units and actuators. These mechatronic devices can be regarded as ideal elements, but working with disturbances. Disturbances in the system can be different, not only because of noises and signal offsets but also because of data processing delays.

It is interesting that delay effects in data processing can be also caused by actions of non-linear elements having characteristics with the dead zone. In mechanical systems such behavior is well recognized as clearance or backlash. Transmission of signals through the dead zone element can result not only in their distortion but also in their delay. Delay effects are especially visible for signals which appear suddenly. For example [54], when the element non-linear characteristic $y(x)$ has a piecewise linear form with the dead zone $y(x) = 0$ for $|x| < a$, and the input signal $x(t) = kt1(t)$, then the output signal

$y(t) = kt1(t - \tau)$, where time delay $\tau = a/k$ (here the parameters a, k – positive constants, $1(t)$ – Heaviside function). This is expressed in Fig. 5.

This work does not propose specific technical solutions for the lane change control system. However, the theoretical considerations to follow below, and the simulation results presented in Section 4 represent a basis for the development and implementation of future vehicle controllers.

Simplified theoretical analyses are presented first. They relate to disturbances occurring in the transposition phase, i.e. in the $Y(t)$ measured signal. In these studies simple transmittance models are used also for real object description. In this way, only the effects of disturbances will be exposed.

3.1. Signal noises and offsets

The block diagram used in these studies can be presented as follow:

The mathematical model defined in the operator's field corresponding to the diagram in Fig. 6 is a system of linear algebraic equations. From these equations one can calculate the formula which expresses error $\Delta Y(s)$ as a linear function of $D_Y(s)$

$$\Delta Y(s) = \frac{R_Y(s)G_{Y\delta}(s)}{1 + R_Y(s)G_{Y\delta}(s)} D_Y(s) \tag{23}$$

This formula indicates that the application of a negative feedback in the regulation structure leads to reduce the error signal. Note that for the extremely reduced version of the transfer function $G_{Y\delta}(s)$ and for the PD regulator this formula has form (24) of a filtering element (a very important feature):

$$\Delta Y(s) = \frac{VG_{\Omega\delta_0} K_{PD}(1 + T_{PD}s)}{s^2 + VG_{\Omega\delta_0}(1 + T_{PD}s)} D_Y(s) \tag{24}$$

We can now analyse two cases:

1. Disturbance caused by noise without non-zero average value.
2. Disturbance caused by a non-zero offset.

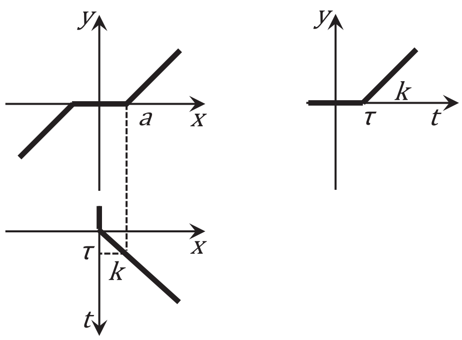


Fig. 5. The effect of the time delay for the signal $x(t)$ which is processed to the form $y(t)$ in the element having a piecewise linear characteristic $y(x)$ with the dead zone.

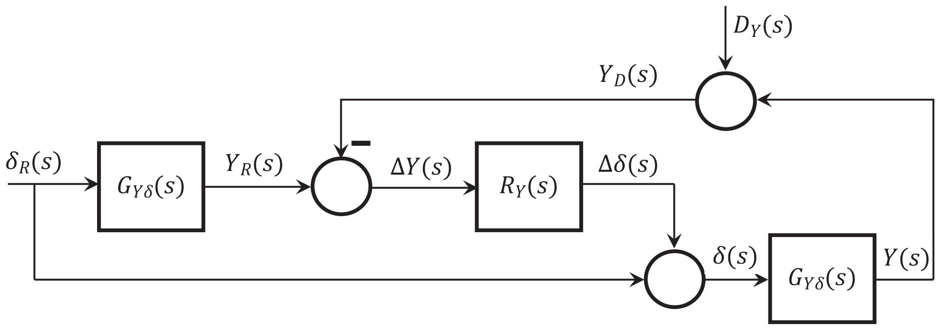


Fig. 6. Block diagram expressing signal disturbances of $y(t)$, with $D_Y(s)$ – the transform of the disturbance signal.

In the 1st case, the noise can be treated as a sum of sinusoidal components. For an individual component its Laplace transform is:

$$L(A_i sin(\omega_i t)1(t)) = A_i \frac{\omega_i}{s^2 + \omega_i^2} \tag{25}$$

A detailed frequency analysis of characteristics $|\Delta Y(\omega)|$ (amplitude) and arg $\Delta Y(\omega)$ (phase) with the help of Bode plots shows quite entangled dependencies of the characteristic frequencies and the model parameters. Such analysis is workable and useful for given values of system parameters.

Important property can be presented by simply calculating the steady state solution that would occur after the interruption of disturbances. Using the fundamental relation of operator calculus, one obtains:

$$\lim_{t\to\infty}(\Delta Y(t)) = \lim_{s\to 0}(s\Delta Y(s)) = 0 \tag{26}$$

It means that regulatory actions can significantly reduce the impact of the noise without the non-zero average value.

In the 2nd case, the disturbance signal has an offset D_{Y_0}. Offset type signal $D_Y(t)$ is regarded here as the step function. We thus have:

$$L(D_{Y_0}1(t)) = \frac{D_{Y_0}}{s} \tag{27}$$

Now it is clear that

$$\lim_{t\to\infty}(\Delta Y(t)) = \lim_{s\to 0}(s\Delta Y(s)) = D_{Y_0} \neq 0 \tag{28}$$

It means that regulatory actions will not reset the control error caused by the offset in the measured $Y(t)$ signal.

3.2. Signal delay effects

The block diagram used in these studies can be presented as follows:

Here the transfer function $G_D(s)$ expresses the action of a delay type disturbance. It has the form:

$$G_D(s) = e^{-s\tau} \tag{29}$$

where: τ – delay time

The mathematical model defined in the operator's field corresponding to the diagram in Fig. 7 is a system of non-linear algebraic equations with exponent functions. From these equations one calculates the formula which expresses error $\Delta Y(s)$ as a function of $\delta_R(s)$:

$$\Delta Y(s) = \frac{(1 - e^{-s\tau})G_{Y\delta}(s)}{1 - (1 - e^{-s\tau})G_{Y\delta}(s)R_Y(s)}\delta_R(s) \tag{30}$$

For further analysis we assume, as before, that the model has a reduced form, the regulator is of the PD type, and the reference signal $\delta_R(t)$ is of a "bang-bang" type. So in this case

$$\delta_R(s) = \frac{\delta_0 \left(1 - e^{-sT}\right)^2}{s} \text{(Laplace transform of the "bang-bang" type function)} \tag{31}$$

For the steady state signal (if it exists) we can calculate, using de l'Hôpital theorem, the following relationship:

$$\lim_{t\to\infty}(\Delta Y(t)) = \lim_{s\to 0}(s\Delta Y(s)) = \lim_{s\to 0} \frac{(1 - e^{-s\tau})VG_{\Omega\delta_0}\delta_0\left(1 - e^{-sT}\right)^2}{s^2 - (1 - e^{-s\tau})VG_{\Omega\delta_0}K_{PD}(1 + T_{PD}s)}$$
$$= \lim_{t\to\infty} VG_{\Omega\delta_0}\delta_0 \frac{e^{-s\tau}\left(1 - e^{-sT}\right) + (1 - e^{-s\tau})e^{-sT}}{2s - VG_{\Omega\delta_0}K_{PD}(e^{-s\tau}(1 + T_{PD}s) + (1 - e^{-s\tau})T_{PD})} = 0 \tag{32}$$

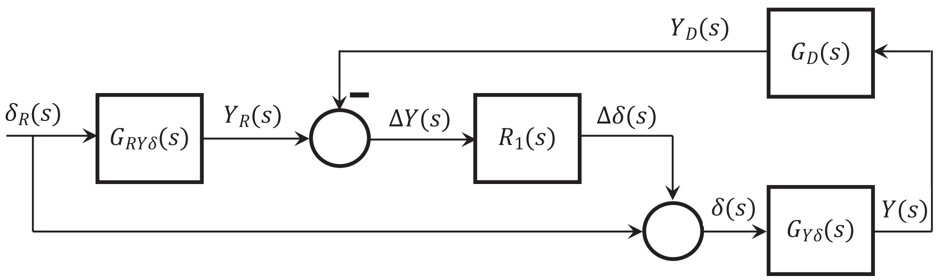


Fig. 7. Block diagram expressing delay effects in the $y(t)$ signal.

This result is very promising. However, it is well-known that control systems with delays are generally difficult to get stabilized, especially when the reference and actual models of the controlled object are significantly different.

Based on the above, to analyze our control system with delays, simulation based studies are necessary.

4. Disturbances in the control system – Numerical investigations

Investigations of the lane change control system have been carried out by extensive simulation tests using a software package developed by the authors. In these tests the controller model controlled a lane-change manoeuvre of a virtual vehicle model build with full detail (3D, multi-body, non-linear model of a two-axle truck of medium load capacity). The vehicle model requires around 200 parameters. Note that the reference model used by the controller uses only 7 parameters. Thanks to many road and stand tests the vehicle model was successfully validated, for a large selection of datasets describing the controller, the vehicle, and road conditions.

The simulation-based sensitivity analysis of mathematical models concerns calculations of signals and integral indexes that express differences between models being compared, while subjected to the same input signals (Fig. 8).

$$\text{Sensitivity index}: W_x = 100\frac{\int_0^{\tau}(x_1(t) - x_2(t))^2 dt}{\int_0^{\tau}(x_1(t))^2 dt} \tag{33}$$

Introducing disturbances and errors to controlled signals (the lateral displacement $Y(t)$ and the yaw angle signals $\Psi(t)$ we can observe their influence on the results of simulations and calculate their sensitivity indexes W_Y and W_Ψ, as well as indexes for other signals, such as $W_{\delta H}$. In these comparative simulations the nominal model (Model 1) is set for perfect measured signals while the changed model (Model 2) is the model with signal disturbances and errors (Fig. 9).

Of course, both models have the same reference steering signal $\delta_{HR}(t)$ of the "bang-bang" type. Disturbances and errors of signals (due to noise, offset and delay) are expressed as follows:

$$Y_D(t) = D_Y(t) + Y(t - \tau) \tag{34}$$

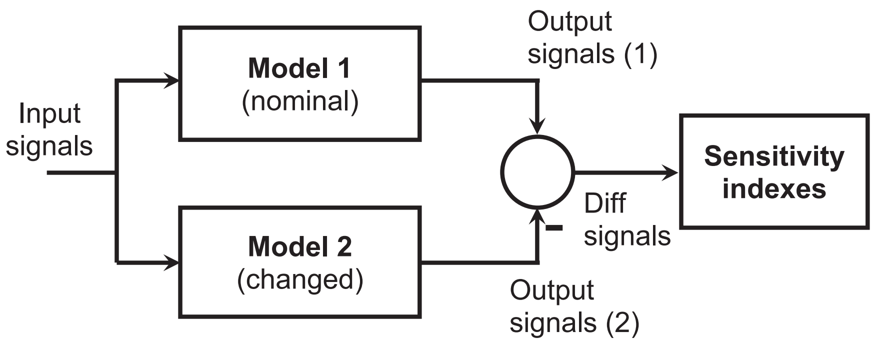


Fig. 8. General schematic diagram of sensitivity analysis based on numerical investigations.

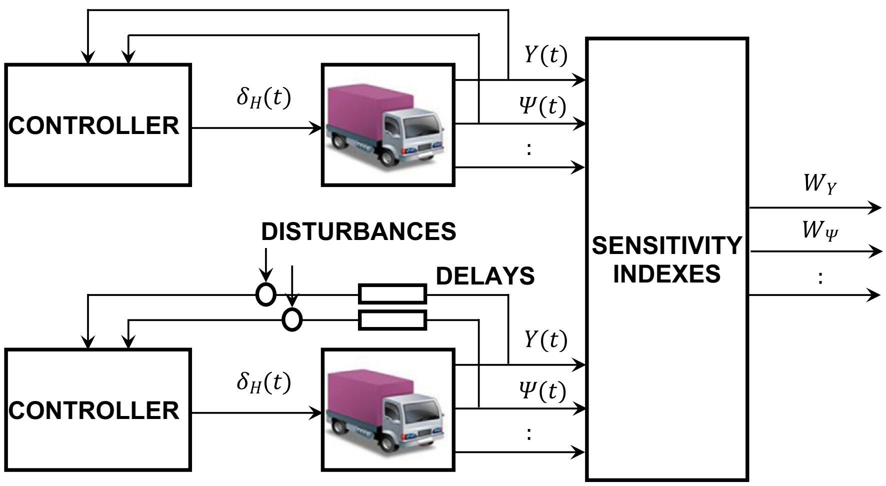


Fig. 9. Schematic diagram of sensitivity analysis of the lane change system based on simulations.

Table 1
The model data.

Model data	Unloaded vehicle	Loaded vehicle
Transverse displacement given Y_0[m]	3	3
Vehicle yaw angle with respect to the road centre line Ψ_0[deg]	8.76	8.76
The steering angle of the front wheels δ_0[deg]	2.24	2.06
The set time to hold the steering angle T[s]	1.01	1.01
Longitudinal velocity of the vehicle (constant) in the local coordinate system V[km/h]	70	70
Vehicle mass m[kg]	5300	11,300
Vehicle inertia moment with respect to the vertical axis passing through its mass centre J[kgm/s^2]	18,000	39,800
Distances from vehicle front wheel axes from the projection of the point representing the vehicle mass centre a[m]	1.681	2.437
Distances from vehicle rear wheel axes from the projection of the point representing the vehicle mass centre b[m]	2.199	1.443
Cornering stiffness for the centres of the front wheel axes k_A[kgm/rd]	236,200	314,900
Cornering stiffness for the centres of the rear wheel axes k_B[kgm/rd]	237,700	625,500

where

$$D_Y(t) = D_{YN}Noise(t) + D_{YO} \tag{35}$$

$$\Psi_D(t) = D_\Psi(t) + \Psi(t-\tau) \tag{36}$$

where

$$D_\Psi(t) = D_{\Psi N}Noise(t) + D_{\Psi O} \tag{37}$$

with D_{YN}, $D_{\Psi N}$ – magnitudes of noise signals, $Noise(t)$ – standard white noise signal, D_{YO}, $D_{\Psi O}$ – offsets.

In these simulations, the modified algorithm of the controller was used. The stabilization regulator started in $t = 1.5T$.

The model data are presented in Table 1 (they are all in SI units in a computer program).

$D_{YN} = 0.2$m, $D_{YO} = \pm 0.2$m, $D_{\Psi N} = 0.1$deg, $D_{\Psi O} = \pm 1.0$deg, $\tau \subset \{0.03, 0.06, 0.09\}$s (the same for both signals).

The assumed values of reference signals parameters indicate that the system is tested also in very challenging conditions of vehicle motion (critical motion state, where linear properties of lateral dynamics can no longer be applied).

Representative exemplary results of simulation studies are presented below, followed by the discussion

5. Discussion

The results of a series of simulations of lane change (left turn) taking into account various values of measurement errors and interferences (delays τ, noises D_{YN}, $D_{\Psi N}$ and offsets D_{YO}, $D_{\Psi O}$) have been presented in the charts. In the simulations, the prescribed driving maneuver was carried out in the shortest possible time and on the shortest possible distance resulting from the prevailing road conditions and the vehicle traffic condition (wet road, adhesion coefficient μ = 0.3, car speed V = 70 km/h). This means that the lane change took place in extreme traffic conditions with a low stability margin. The two most common loading conditions of the truck (unloaded and fully loaded) were analyzed. This choice was interesting to the authors because our earlier studies showed that the load condition of the vehicle clearly affects the vehicle steering characteristics, especially at high lateral acceleration levels. Unloaded vehicle with increasing lateral acceleration approaching its allowed limit value on a given surface, rapidly changes the steering characteristics from understeer to oversteer, which quickly leads to the loss of its directional stability. This fact pointed to the possibility of problems in the automatic control of such unloaded vehicle in threshold traffic conditions. On the other hand, a fully loaded vehicle maintains understeer characteristics throughout the entire lateral acceleration range. Its understeer increases rapidly as it approaches the lateral acceleration limit. This property makes it significantly more resistant to the danger of losing its directional stability.

Fig. 10 shows the influence of the value of the time delay of the lateral displacement Y and the yaw angle Ψ signals on the quality of the lane change carried out by the truck, and the steering wheel rotation δ_H necessary for this maneuver to be carried out correctly. A fully loaded car turned out to be very insensitive to increasing the delay time τ = 0–0.09 s. Only increasing the delay time to the value of τ > 0.2 s made it impossible to perform the lane change maneuvers. The unloaded car was much more responsive to increasing the time delay and for τ = 0.09 s it lost the directional stability. Significant differences in the values of the W_x ($W_{\delta H}$, W_Y, W_Ψ) indexes for the unloaded and loaded car for different time delays τ are shown in Fig. 11.

Figs. 12 and 13 show in tables and graphs the impact of the noise in $Y(t)$ and $\Psi(t)$ signals on W_x indicators for different values of the time delay τ. Changes in the delay τ = 0–0.09 s, noise D_{YN} = 0–0.2 m (Fig. 12) and $D_{\Psi N}$ = 0–1.0 deg (Fig. 13) were considered. The impact of each type of the noise on the lane change process was considered separately. It was found that the impact of the noise (with values in the considered ranges) on the values of the W_x indicators was practically negligible as compared to the effect of the signal delay τ. This impact was greater for the unloaded vehicle. The above conclusions are consistent with the results of theoretical considerations carried out in Section 3 (expression (26)).

Figs. 14 and 15 show in tables and graphs the impact of the offsets in $Y(t)$ and $\Psi(t)$ signals on W_x indicators with the time delay τ varying. The changes of the delay τ = 0–0.09 s, the offsets D_{YO} = −0.2–0.2 m (Fig. 14) and $D_{\Psi O}$ = −1.0–1.0 deg were

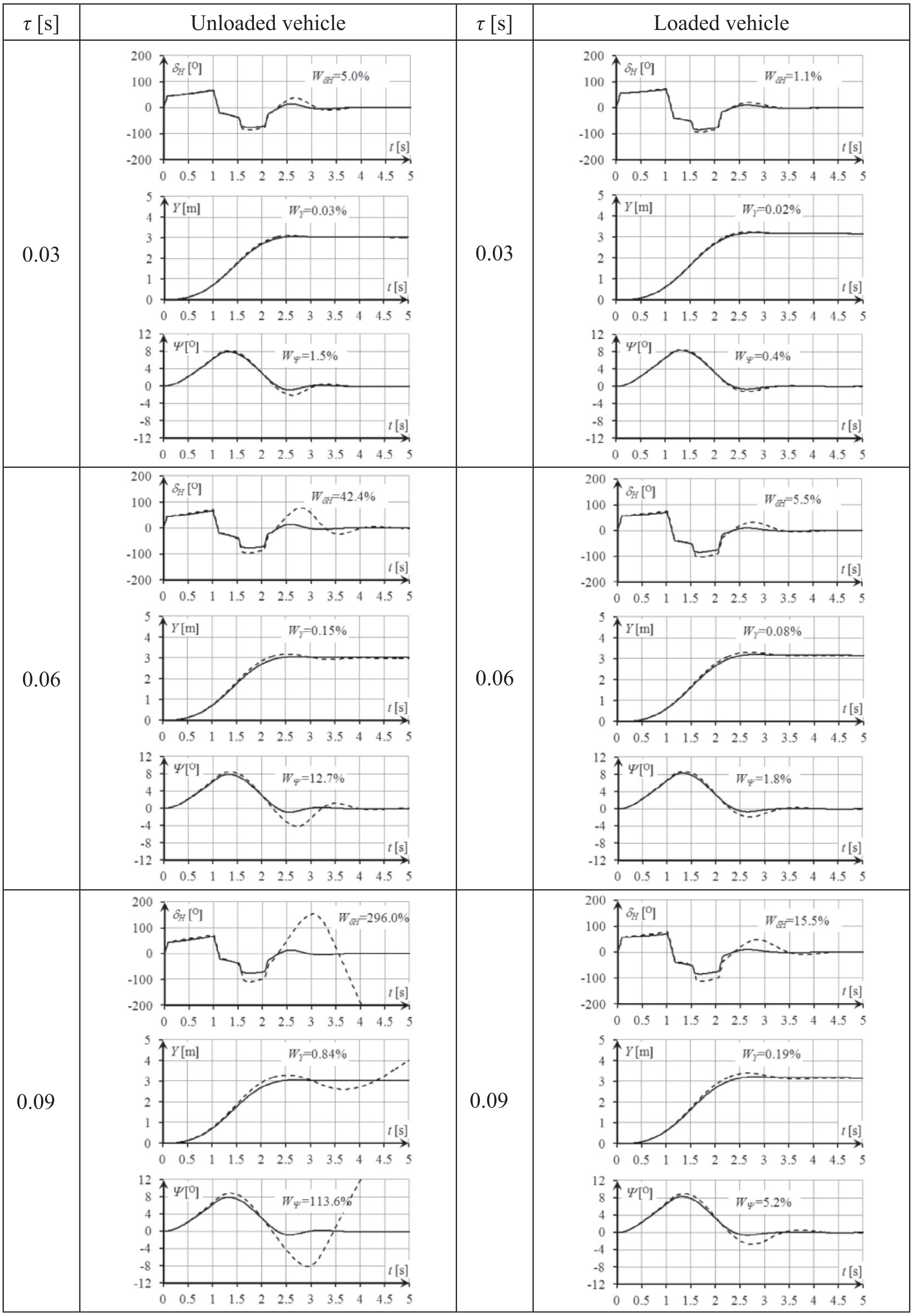


Fig. 10. Results of simulations when measured signals have been delayed (here the noises and offsets are absent). Notation: solid lines for the system without delays, dashed lines for the system with delays.

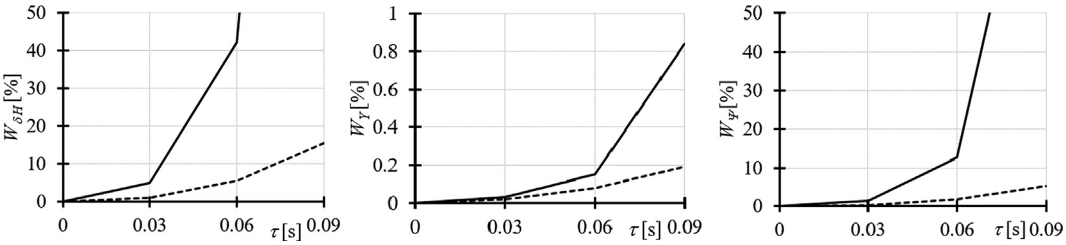


Fig. 11. Dependence of the sensitivity indexes on the time delay (here the noises and offsets are absent). Notation: solid lines for the unloaded vehicle, dashed lines for the loaded vehicle.

considered. The impact of each type of the offsets on the course of the lane change process was considered separately. The influence of the offsets D_{YO} and $D_{\Psi O}$ on the courses was clear and much more significant than the effect of the time delay τ. Of course, this also manifested itself in changes in the values of W_x indexes.

For the unloaded vehicle, the offset $D_{YO} < 0$ proved to be particularly dangerous. With $D_{YO} = -0.2$ m and $\tau = 0.06$ s, the vehicle lost its directional stability. For $\tau = 0.03$ s, the unloaded vehicle changed the lane correctly, but that required the steering system action with more complexity and with higher amplitude of the signal $\delta_H(t)$. It caused the maneuver time to be additionally extended and resulted in exceeding the desired lateral displacement of the vehicle by >0.5 m, as well as producing large angular deviations. Of course, W_x indexes had much higher values than for the zero offset t. It is interesting that the offset $D_{YO} > 0$ practically did not affect the values of the W_x indexes (Fig. 14). The loaded vehicle reacted much less to the change of the offset D_{YO}, but also in this case a greater impact of the $D_{YO} < 0$ shift than the $D_{YO} > 0$ shift could be seen. Shifting $D_{YO} \neq 0$ causes the vehicle after a lane change to move parallel to the road axis, but above (($D_{YO} > 0$) or below ($D_{YO} < 0$)) of the target line. The difference in the lateral displacement Y relative to the set point Y_0 directly depends on the value of the D_{YO} shift.

The offset $D_{\Psi O}$ also caused different effects when it was positive or negative. The unloaded vehicle lost directional stability at the time delay $\tau = 0.06$ s and the shift of $D_{\Psi O} = 1.0$ deg. When $D_{\Psi O} = -1.0$ deg the lane change process was stable but the most of the W_x indexes had higher values than at the zero shift (Fig. 15). The loaded vehicle was definitely less sensitive to the $D_{\Psi O}$ yaw angle offset than the unladed vehicle. All W_x indexes calculated for this vehicle had clearly smaller values. Most of the indicators calculated for $D_{\Psi O} = 1$ deg had smaller values than for $D_{\Psi O} = -1$deg. Shifting the yaw angle $D_{\Psi O} \neq 0$ always caused negative effects, because the vehicle after the lane change maneuver drove straight ahead but at a certain angle to the road axis going left ($D_{\Psi O} > 0$) or right $D_{\Psi O} < 0$) from the target lane.

It should be noted that also in the case of D_{YO} and $D_{\Psi O}$ offsets, their impact on the quality of the lane change maneuver is consistent with the results of theoretical considerations carried out in Section 3 (expression (28)).

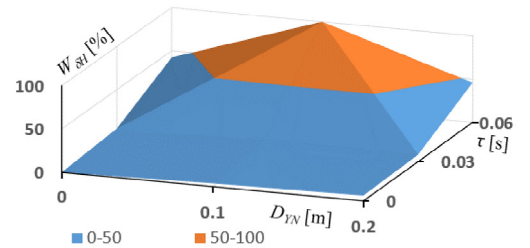
It is worth explaining why the D_{YO} and $D_{\Psi O}$ shifts, depending on whether they have a negative or positive sign, significantly affect the process of vehicle lane changing. Figs. 16 and 17 are helpful in explaining this. It should be noted that these figures present examples of simulation results used to fill the tables and making the charts in Figs. 11–15.

Fig. 16 shows the waveforms of variables when changing the right lane to the left for the delay of $\tau = 0.06$ s and the offset $D_{YO} = \pm 0.2$ m. The $D_{YO} = -0.2$ m offset means that the adjustment controller gets the information that incorrectly specifies the lateral displacement of the vehicle's center of mass, reducing it by 0.2 m as compared to the actual displacement. The transposition regulator therefore increases the steering wheel rotation angles more than necessary, which leads to excessive lateral displacement and deviation angles. After switching on the stabilization regulator (at $t = 1.5$ T), it already receives correct information about a too large difference between the target and the actual deviation angle. As a result, the steering wheel is turned by a value that causes high lateral acceleration and large vehicle yaw velocity. The unloaded vehicle with a low stability reserve loses directional stability despite significant and fast steering wheel rotations to the left and to the right. The loaded vehicle with a larger stability reserve successfully completes the lane change, but this requires the steering wheel to be turned by a larger angle. This results in greater than prescribed lateral displacement of the vehicle both during and after the completion of the maneuver. The opposite is true for the offset $D_{YO} = 0.2$ m. In this case, the adjustment regulator is informed about the lateral displacement of the vehicle greater by 0.2 m than its real value. The steering wheel turns have therefore smaller angles and smoother profiles, and resulting lateral accelerations and yaw speeds are less than allowed. The unloaded and loaded vehicle performs the lane change maneuver correctly with sufficient stability. The profiles of the analyzed signals do not differ much (for the unloaded vehicle more) than the runs generated for $D_{YO} = 0$.

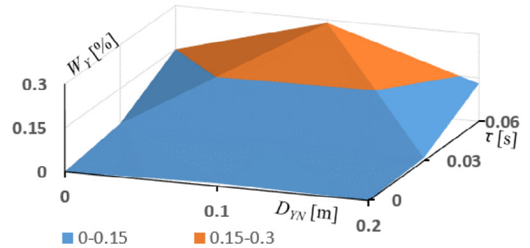
Fig. 17 presents the time profiles of the same quantities as in Fig. 16 recorded when changing the lane from right to the left for $\tau = 0.06$ s and $D_{\Psi O} = \pm 1.0$ deg. In this case, an incorrect measurement of the yaw angle only affects the operation of the stabilization regulator. When $D_{\Psi O} = 1.0$ deg, the stabilization regulator, immediately after switching on, receives an incorrect information about the value of the yaw angle, which is larger by 1 deg than its real value. This causes larger than necessary rotation of the steering wheel, which in the case of an unloaded vehicle leads to the loss of lateral stability. Also, in the case of a laden vehicle it hinders and extends its stabilization. After the maneuver, the car moves obliquely to the lane axis and goes towards its right edge. When $D_{\Psi O} = -1.0$ deg, the steering wheel movements provided by the regulator are smaller than nec-

Unloaded vehicle	Loaded vehicle
τ (0s, 0.09 s); D_{YN} (0 m,0.2 m) $D_{\psi N}$=0, D_{YO}= 0, $D_{\psi O}$= 0	τ (0 s, 0.09 s); D_{YN} (0 m,0.2 m) $D_{\psi N}$=0, D_{YO}= 0, $D_{\psi O}$= 0

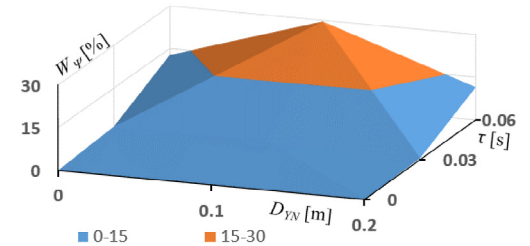
Unloaded vehicle



	$W_{\delta H}$ [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0.0	5.0	42.2	us
0.1	3.7	8.4	100	us
0.2	6.1	8.0	46	us

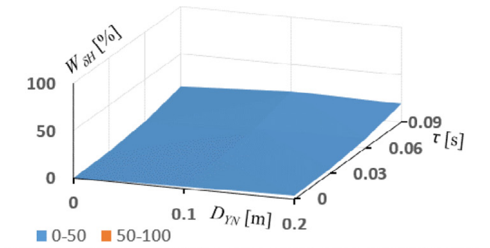


	W_Y [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0.0	0.03	0.15	us
0.1	0.01	0.02	0.29	us
0.2	0.001	0.01	0.13	us

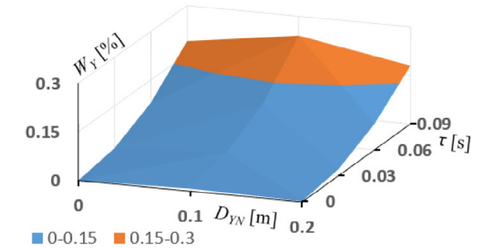


	W_Ψ [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0.0	1.5	12.7	us
0.1	0.1	1.4	29.6	us
0.2	0.05	0.5	11.5	us

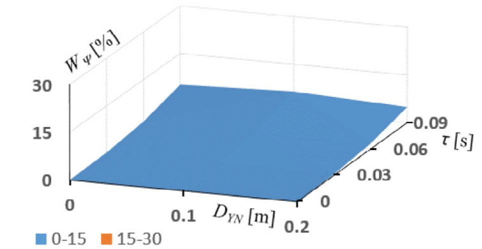
Loaded vehicle



	$W_{\delta H}$ [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0	1.1	5.5	15.5
0.1	2.0	3.9	9.5	21.5
0.2	3.7	4.9	9.5	19.5



	W_Y [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0	0.02	0.08	0.19
0.1	0.008	0.04	0.11	0.24
0.2	0.002	0.02	0.08	0.18

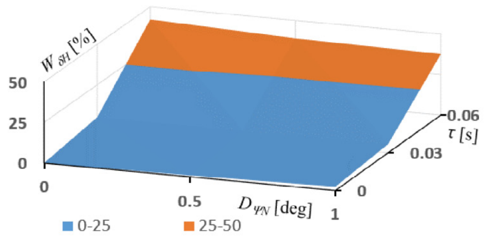


	W_Ψ [%]			
D_{YN} [m] \ τ [s]	0.0	0.03	0.06	0.09
0.0	0	0.36	1.8	5.2
0.1	0.04	0.6	2.4	6.4
0.2	0.02	0.35	1.7	4.9

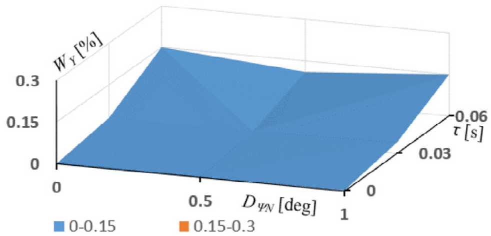
Fig. 12. Dependence of sensitivity indexes on the noise and the delay time in the signal $Y(t)$.

Unloaded vehicle	Loaded vehicle
τ (0 s, 0.09 s); D_{YN}= 0, $D_{\Psi N}$ (0deg,1deg), D_{YO}= 0, $D_{\Psi O}$=0	τ (0s, 0.09s); D_{YN}= 0, $D_{\Psi N}$ (0 deg,1 deg), D_{YO}= 0, $D_{\Psi O}$= 0

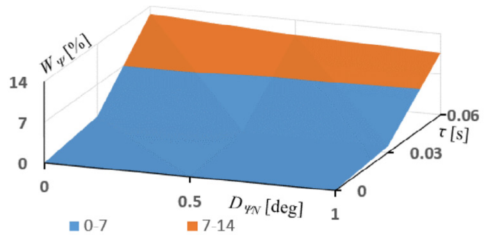
Unloaded vehicle



	$W_{\delta H}$ [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0.0	5.0	42.2	us
0.5	0.9	4.8	38.9	us
1.0	2.3	5.3	37.7	us

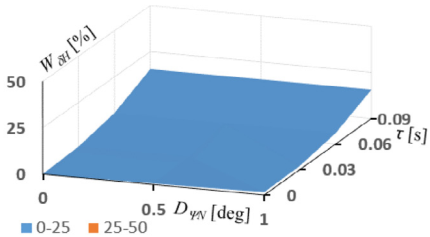


	W_Y [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0.0	0.03	0.15	us
0.5	0.002	0.03	0.11	us
1.0	0.003	0.04	0.15	us

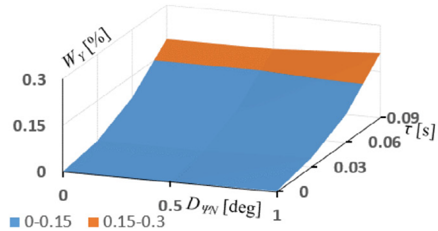


	W_ψ [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0.0	1.5	12.7	us
0.5	0.02	1.2	11.4	us
1.0	0.06	1.1	10.7	us

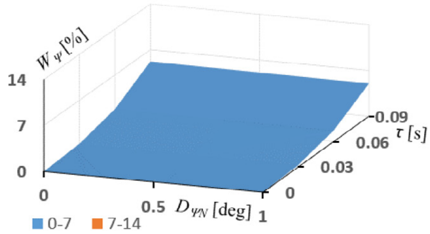
Loaded vehicle



	$W_{\delta H}$ [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0	1.1	5.5	15.5
0.5	0.57	1.4	5.7	15.6
1.0	1.5	2.0	5.9	15.6



	W_Y [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0	0.02	0.08	0.19
0.5	0.001	0.02	0.08	0.19
1.0	0.003	0.03	0.09	0.21

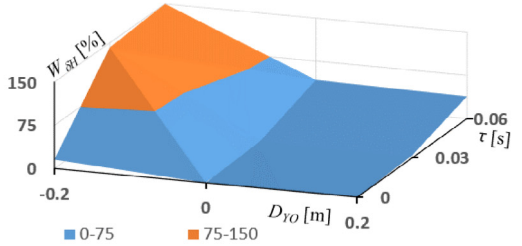


	W_Ψ [%]			
τ [s] / $D_{\Psi N}$ [deg]	0.0	0.03	0.06	0.09
0.0	0	0.36	1.8	5.2
0.5	0.01	0.31	1.7	5.1
1.0	0.03	0.29	1.7	5.0

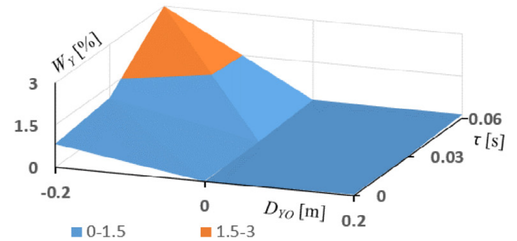
Fig. 13. Dependence of sensitivity indexes on the noise and the delay time in the signal $\Psi(t)$.

Unloaded vehicle	Loaded vehicle
τ (0 s, 0.09 s); $D_{YN} = 0$, $D_{\Psi N} = 0$, D_{YO} (-0.2 m, 0.2 m), $D_{\Psi O} = 0$	τ (0 s, 0.09 s); $D_{YN} = 0$, $D_{\Psi N} = 0$, D_{YO} (-0.2 m, 0.2 m), $D_{\Psi O} = 0$

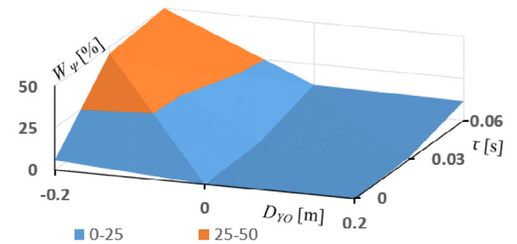
Unloaded vehicle



D_{YO} [m] \ τ [s]	$W_{\delta H}$ [%] 0.0	0.03	0.06	0.09
-0.2	15.4	142	us	us
0.0	0.0	5.0	42.2	us
0.2	0.1	4.2	38.6	us

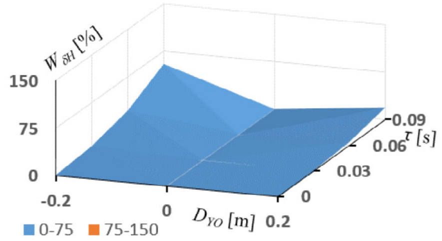


D_{YO} [m] \ τ [s]	W_{Y} [%] 0.0	0.03	0.06	0.09
-0.2	0.83	1.1	us	us
0.0	0.0	0.03	0.15	us
0.2	0.04	0.03	0.13	us

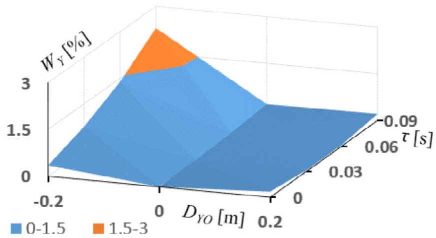


D_{YO} [m] \ τ [s]	W_{Ψ} [%] 0.0	0.03	0.06	0.09
-0.2	5.8	46.2	us	us
0.0	0.0	1.5	12.7	us
0.2	0.04	1.3	11.5	us

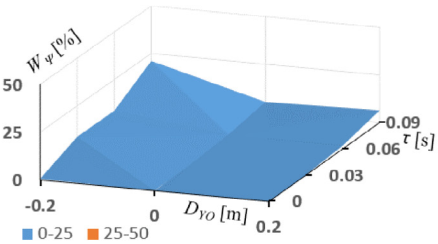
Loaded vehicle



D_{YO} [m] \ τ [s]	$W_{\delta H}$ [%] 0.0	0.03	0.06	0.09
-0.2	0.77	6.3	22.0	54.7
0.0	0.0	0.03	0.06	0.09
0.2	0.29	0.6	5.5	19.4



D_{YO} [m] \ τ [s]	W_{Y} [%] 0.0	0.03	0.06	0.09
-0.2	0.35	0.77	1.39	2.3
0.0	0	0.02	0.08	0.19
0.2	0.18	0.04	0.04	0.21



D_{YO} [m] \ τ [s]	W_{Ψ} [%] 0.0	0.03	0.06	0.09
-0.2	0.54	8.8	8.7	21.1
0.0	0	0.36	1.8	5.2
0.2	0.22	0.2	1.7	6.4

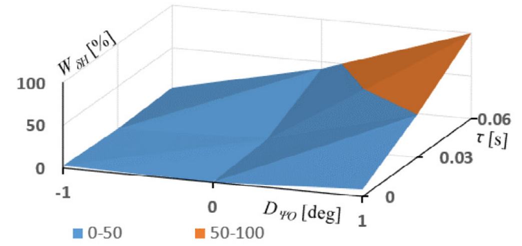
Fig. 14. Dependence of sensitivity indexes on the offset and the delay time in the signal $Y(t)$.

Unloaded vehicle

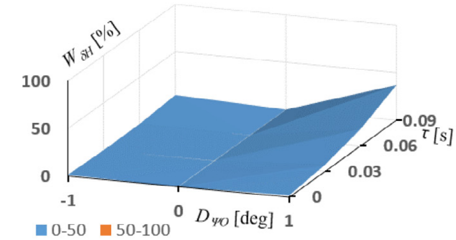
τ (0 s, 0.09 s); D_{YN}= 0, $D_{\Psi N}$= 0, D_{YO}= 0, $D_{\Psi O}$ (-1.0 deg, 1.0 deg)

Loaded vehicle

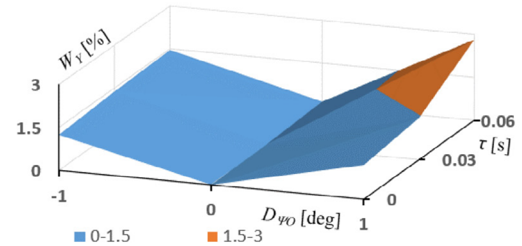
τ (0 s, 0.09 s); D_{YN}= 0, $D_{\Psi N}$= 0, D_{YO}= 0, $D_{\Psi O}$ (-1.0 deg, 1.0 deg)



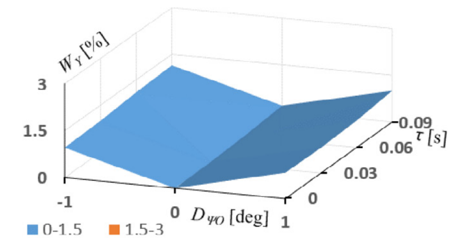
	$W_{\delta H}$ [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	2.7	0.4	3.4	us
0.0	0.0	5.0	42.2	us
1.0	8.1	49.7	us	us



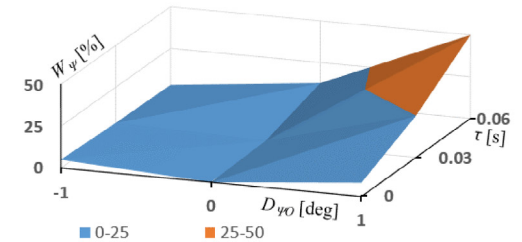
	$W_{\delta H}$ [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	1.3	0.3	0.9	4.5
0.0	0.0	0.03	0.06	0.09
1.0	1.9	7.1	17.9	37.1



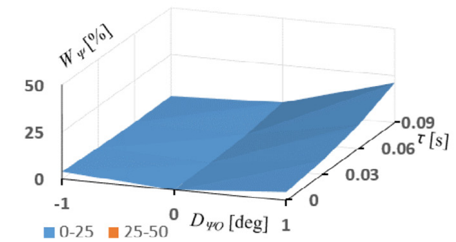
	W_Y [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	1.24	1.38	1.46	us
0.0	0.0	0.03	0.15	us
1.0	1.19	1.46	us	us



	W_Y [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	0.96	1.0	1.1	1.15
0.0	0	0.02	0.08	0.19
1.0	0.83	0.81	0.86	1.0



	W_Ψ [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	5.0	3.7	3.1	us
0.0	0.0	1.5	12.7	us
1.0	8.2	25.0	us	us



	W_Ψ [%]			
τ [s] / $D_{\Psi O}$ [deg]	0.0	0.03	0.06	0.09
-1.0	3.9	3.1	2.6	2.9
0.0	0	0.36	1.8	5.2
1.0	4.4	7.3	12.5	21.2

Fig. 15. Dependence of sensitivity indexes on the offset and the delay time in the signal $\Psi(t)$.

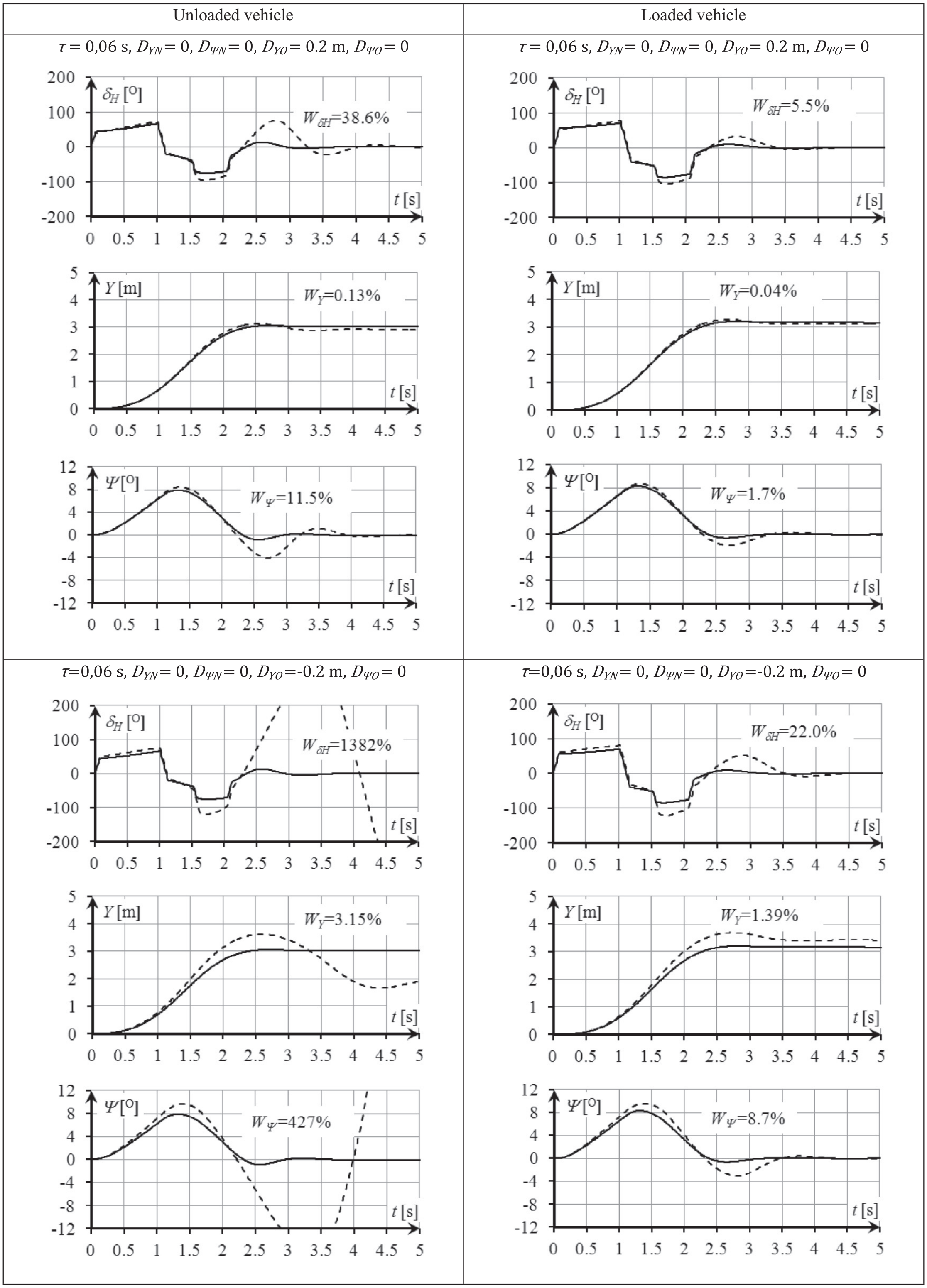


Fig. 16. Results of simulations when measured signals have been delayed and the offset present in $Y(t)$ (no noises). Notation: solid lines for the case without offset, dashed lines – with the offset.

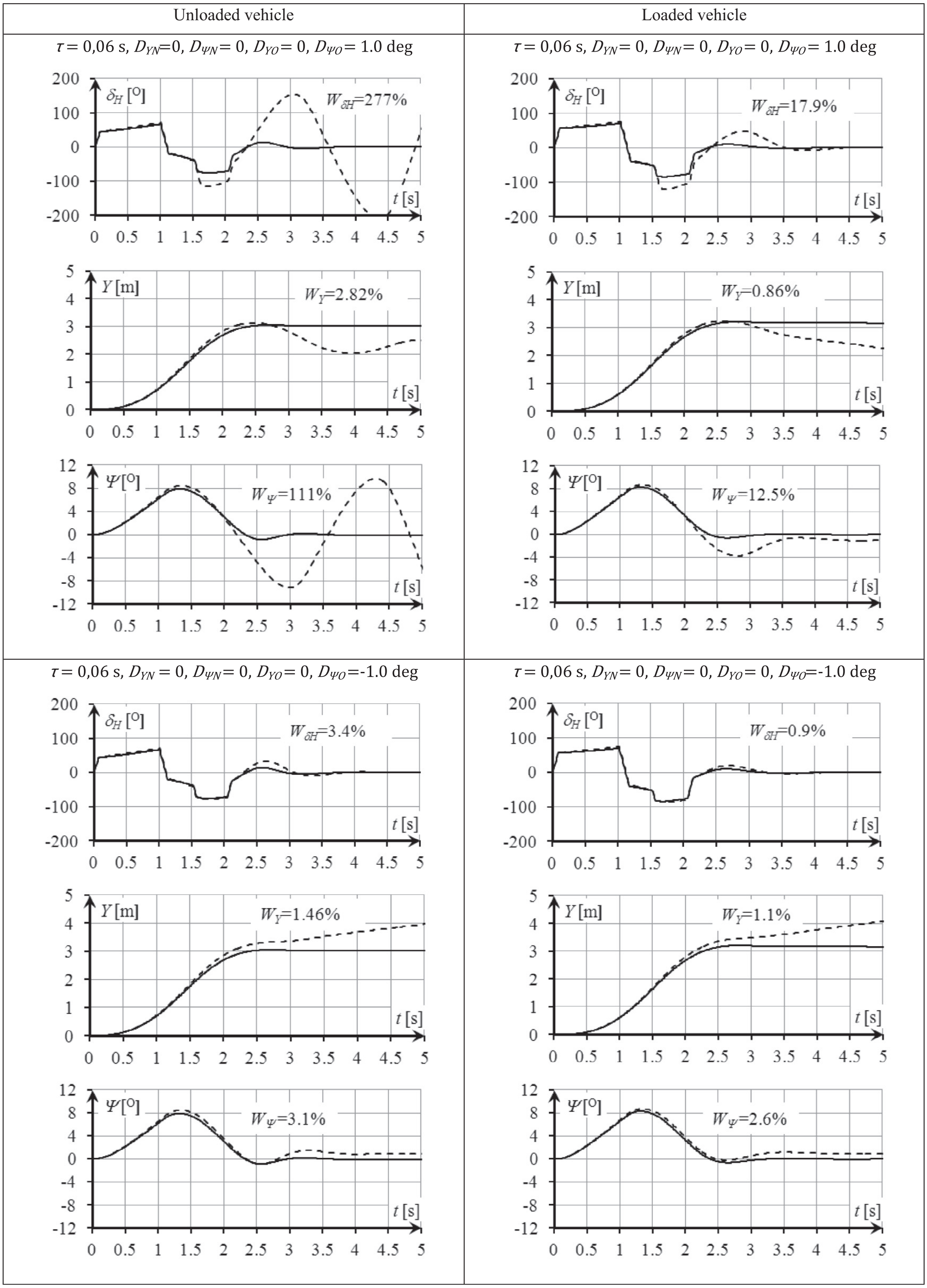


Fig. 17. Results of simulations when measured signals have been delayed and the offset present in $\Psi(t)$ (no noises). Notation: solid lines for the case without offset, dashed lines – with the offset.

essary to make a proper lane change. The maneuver takes place with a greater margin of stability, but the vehicle (either unloaded or loaded) moves into the left lane towards its left edge.

6. Summary

The presented studies show the complexity of the lane change processes and generally confirm the advantages of the automatic control solutions adopted. The simple standard regulators used in the controller, correcting the reference "bang-bang" control signal generated, look to be effective even if the reference model of the vehicle is significantly reduced. This confirms the known effectiveness of negative feedback which allows to mitigate the effects of imperfections of the adopted object model.

The simulation based sensitivity analyses show that in almost all cases the lane change maneuver has been successfully carried out, in spite of the measurement disturbances and errors of the measured transposition signal $Y(t)$ and yaw signal $\Psi(t)$. For the system working without delays, satisfactory results appear when the controller parameters are properly chosen (for given operating conditions), and the measurement offset and noise are not too large. Especially dangerous for automatic control is the offset of a signal. After increasing the measurement offset to a higher level and the noise to a higher amplitude it was observed, for selected operating conditions (lightly loaded lorry and a wet road), that the vehicle was losing directional stability. At the small offset and the low noise levels in the measurement signals, despite wide range changes of operating conditions, the automatically operated vehicle was able to change the lane without losing directional stability.

Time delays of the measured signals are very dangerous for the system. The stability zone depends on vehicle parameters and road conditions. Extremely difficult operating conditions (lightly loaded lorry and a very wet road) were proven especially difficult for stability. For $\tau > 0.2$ s, the system was practically unstable in all conditions.

It can be also stated that the cumulative occurrence of signal disturbances and errors (noise plus offset plus delay) can rule out proper operation of the automatic control system. But a question can be raised whether even an experienced race car driver would be able to cope with such a set of extremely difficult control conditions.

There are several possible ways to avoid problems related to vehicle control during execution of the lane change maneuver:

(1) Increasing the stability reserve for the unloaded vehicle by lowering the value of the steering wheel angle set value – however, this unfortunately leads to an extension of the travel distance needed for the maneuver.
(2) Enabling two controllers to work simultaneously by switching on the stabilization controller at an appropriately chosen time instant [53].
(3) Integrating the operation of the automatic steering system controller with the ESC (Electronic Stability Control) system, allowing additional stabilization by the brake system.

The authors of this article continue working on improving the developed automatic lane change control system by taking into account all of the above suggested ideas in their research.

CRediT authorship contribution statement

Mirosław Gidlewski: Supervision, Software, Investigation, Validation, Writing - original draft. **Jerzy Jackowski:** Project administration, Resources, Validation. **Leszek Jemioł:** Software, Investigation, Visualization, Data curation. **Dariusz Żardecki:** Conceptualization, Methodology, Formal analysis, Investigation, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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